CS-210 Discrete Mathematics

Problem Set 4

1. Find at least three different sequences beginning with the terms $1, 2, 4$ whose terms are generated with a simple formula or rule. Assuming that your formula or rule is correct, determine the next 5 terms in each case.

2. Let a_n be the nth term of the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots$, constructed by including the integer k exactly k times. Show that $a_n = \lfloor \frac{1}{2} + \sqrt{2n + \frac{1}{4}} \rfloor$.

3. Show that if a_n denotes the nth positive integer that is not a perfect square, then $a_n = n + \{\sqrt{n}\}$ where $\{x\}$ denotes the integer closest to the number x.

4. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequences that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

   (a) $3, 6, 11, 18, 27, 38, 51, 66, 83, 102, \ldots$

   (b) $7, 11, 15, 19, 23, 27, 31, 35, 39, 43, \ldots$

   (c) $1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, \ldots$

   (d) $0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, \ldots$

   (e) $1, 3, 15, 105, 945, 10395, 135135, 2027025, 34459425, \ldots$

   (f) $1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, \ldots$

   (g) $2, 4, 16, 256, 65536, 4294967296, \ldots$

5. What are the values of these sums?

   (a) $\sum_{i=1}^{5} (i + 1)$

   (b) $\sum_{i=0}^{4} (-2)^i$

   (c) $\sum_{i \in \{1,3,5,7\}} i^2$

   (d) $\sum_{i \in \{1,3,5,7\}} \frac{1}{i}$

6. What are the values of these sums?

   (a) $\sum_{i=1}^{4} (i^2 + i)$

   (b) $\sum_{k=1}^{4} k^2 + \sum_{k=1}^{4} k$

(c) $\sum_{i=0}^{4}(3i^2+2i)$

(d) $3\sum_{k=0}^{4}k^2+2\sum_{k=0}^{4}k$

7. Let $|r|<1$ be a real number. Evaluate

$$\sum_{i=0}^{\infty} ir^i$$

8. Find a formula for $\sum_{k=0}^{m}\lfloor\sqrt{k}\rfloor$ where m is a positive integer.