Lahore University of Management Sciences

EE 563 / MATH 325: Convex Optimization

Spring 2024

Course Catalog Description
This course focuses on theory, algorithms and applications of convex optimization. Convex optimization deals with optimization problems where the objective function and the constraints of the problem are both convex. These problems appear in a variety of applications in diverse fields of science and engineering (e.g., statistics, signal/image processing, wireless communications, medical imaging, machine learning, economics, to name a few). Students will be trained to recognize, model/formulate, and solve convex optimization problems. Applications will revolve around medical imaging, big data and machine learning, and statistical (parameter) estimation. The course lectures will be divided into 4 sections: (1) Basics of convex analysis, (2) First-order methods, (3) Duality, and (4) Second-order methods. For advanced topics, we will cover selected problems covering the research themes of the class. Implementation of optimization algorithms will be carried out in MATLAB/Python. **Pedagogical approach:** We will develop a strong motivation to study these tools using applications (and examples) from high-school, college, and undergraduate mathematics and physics followed by learning of high-level concepts and algorithms using intuition and reasoning.

Course Details	
Credit Hours	3
Core	Electrical Engineering, Mathematics
Elective	Computer Science, Economics, Biology, and others
Open for Student Category	BS (needs prior permission) / MS / PhD
Closed for Student Category	Without an undergraduate degree and/or no training/tutelage in Calculus

Course Prerequisite(s)/Co-Requisite(s)	
Pre-requisites	Linear Algebra, MATLAB/Python
Co-requisites	None
Recommended	Linear Systems Theory, Introduction to Analysis, Signals and Systems

Course Offering Details					
Credit Hours	3				
Classes (per week)	2	Duration	75 min each	Timings & Venue	Tue & Thu, 2:00 – 3:15 PM, SBASSE Block 10-201
Recitation (per week)	None	Duration	NA		
Tutorial (per week)	None	Duration	NA		

Course Teaching Methodology				
Main Lectures	Mode	In-Class, On Campus	Duration (per week)	75 minutes
Problem Solving Sessions	Mode	In-Class or Online (Zoom) (based on student request)	Duration (per week)	Open-ended

Personnel Details	
Lead Instructor	**Hassan Mohy-ud-Din, PhD**
Room No.	9-246, Tesla Wing, SBASSE (second floor, Tesla Wing, second right, first right)
Office Hours	appointment by email (hassan.mohyuddin@lums.edu.pk)
Email	hassan.mohyuddin@lums.edu.pk
Telephone	+92 42 3560 8000, Ext 8505
TA	Nehal Ahmed Shaikh (24020001@lums.edu.pk)
TA Office Hours	TBA
Course URL (if any)	LMS Site

Grading Breakup and Policy **(Tentative)**		
Class Quizzes	40%	Can be announced or unannounced. At least 7 Quizzes will contribute towards final grade.
Midterm Exam	30%	Combined and closed book.
Final exam	30%	Combined and closed book.

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Assessed Course Learning Outcomes	
EE – 563	The students should be able to:
CLO1:	understand the basic theory of convex optimization.
CLO2:	recognize and formulate convex optimization problems appearing in a variety of applications.
CLO3:	analyze a convex problem using convex optimization theory and duality theory.
CLO4:	solve optimization problems by using different state-of-the-art algorithms.
CLO5:	use the tools/methods learnt in this course in various (research) applications.

Relation to EE Program Outcomes

EE-563 CLOs	Related PLOs	Levels of Learning	Teaching Methods	CLO Attainment checked in
CLO1	PLO2		Instruction and Assignments	Midterm and Final Exam
CLO2	PLO2		Instruction and Assignments	Midterm and Final Exam
CLO3	PLO2		Instruction and Assignments	Midterm and Final Exam
CLO4	PLO3		Instruction and Assignments	Midterm and Final Exam
CLO5	PLO2, PLO3		-	Midterm and Final Exam

Course Overview

Week No.	Book Chapter or References	Topic	Related CLOs & Additional Remarks
1	Class lecture, handouts, and reading assignment	**Optimization as 'We Know It'** In this lecture I will talk about early encounters with optimization which a student comes across in high-school, college, and undergraduate mathematics and physics without *possibly* knowing (or hearing) about *optimization theory*. **Why do we need to know Linear Algebra?** With three to four real world applications from machine learning, signal and image processing, and statistical inference, I will elaborate the need of learning (some) Linear Algebra tools to study the theoretical and practical aspects of optimization. We will revise the following machinery from Linear Algebra: • Vector and Matrices • Metric space, Inner product space, and norms • Eigen decomposition and Singular Value Decomposition (SVD) • Basic concepts in Topology	CLO1
2 – 3	Class lecture, handouts, and reading assignment	**Fundamentals of Convexity** We will talk about convex sets and convex functions, their intrinsic properties, how to establish convexity, operations preserving convexity, and applications in machine learning, signal and image processing, and statistical inference.	CLO1, CLO2, CLO3
4	Class lecture, handouts, and reading assignment	**Nomenclature of Convex Optimization** We will talk about a generalized formulation of convex optimization problem (with constraints) and the corresponding optimization terminology/nomenclature.	CLO1, CLO2, CLO3
5 – 6	Class lecture, handouts, and reading assignment	**Unconstrained Optimization: Optimality Conditions** We will talk about optimality conditions for unconstrained optimization problems. The following concepts will be discussed: • Local and global optima • First-order optimality conditions • Definiteness of a Matrix (positive (semi) definite, negative (semi) definite, and indefinite matrices) • Second-order optimality conditions • Global optimality conditions • Quadratic Functions	CLO1
7	Class lecture, handouts, and reading assignment	**Least Squares** We will talk about one of the most important (and pervasive) optimization problems of all times. The following concepts will be discussed: • Problem formulation – Unconstrained Least Squares	CLO4, CLO5

		• Properties of Least Squares optimization problem • Condition number and stability • Cholesky Factorization, QR Factorization, and Singular Value Decomposition methods • Regularized Least Squares • Nonlinear Least Squares	
8 – 10	Class lecture, handouts, and reading assignment	**Unconstrained Optimization: Descent Algorithms** We will talk about solving unconstrained problems using Descent methods. The following concepts will be touched upon: • How to pick descent direction and which is the steepest direction • Characterizing descent directions • Step-size selection • Line search: Exact line search, In-exact line search, and Backtracking line search • Armijo condition, Sufficient decrease, and Wolfe conditions • Gauss-Newton Method • Convergence Analysis • Zoutendijk Theorem	CLO4, CLO5
11	Class lecture, handouts, and reading assignment	**Newton Method** In second-order methods, we will study Newton method for unconstrained minimization. We will draw comparisons with Gradient Descent followed by analysis of convergence. We will also study two algorithms and implement in MATLAB: • Pure Newton's Method • Damped Newton's Method	CLO4, CLO5
12	Class lecture, handouts, and reading assignment	**Duality and KKT Conditions** In these lectures will study the role of duality in convex optimization, primal-dual formulation of an optimization problem, strong duality, duality gap, and KKT conditions for optimality. The concept will be elaborated using real-world applications from machine learning, image restoration, and statistical inference. We also solve these problems in MATLAB.	CLO3
13	Class lecture, handouts, and reading assignment	**Conjugate Gradient Method** We will develop and implement the afore-mentioned algorithm in MATLAB followed by exploring its convergence properties. Conjugate Gradient algorithm will be used for quadratic and non-quadratic problems.	CLO4, CLO5
14	Class lecture, handouts, and reading assignment	**Advanced Topics** **If time permits,** we will focus on the following themes (in that order): • Accelerated Gradient Method • Proximal Gradient Descent • Projected Gradient Descent • ADMM	CLO4, CLO5

Note:

- We will spread this content over a 14-week semester.
- MATLAB/Python will be a routine feature in this course. Make sure you have MATLAB/Python installed on your computers/laptops.
- It is recommended that students revise important concepts in Linear Algebra.

Examination Detail	
Midterm Exam	**Yes/No:** Yes **Combine Separate:** combined **Duration:** 2 – 3 hours **Preferred Date:** early in the mid-term week **Exam Specifications:** Closed book, closed notes, no help sheets, all relevant formulas will be provided.
Final Exam	**Yes/No:** Yes **Combine Separate:** combined **Duration:** 3 hours **Exam Specifications:** Closed book, closed notes, no help sheets, all relevant formulas will be provided.

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Textbook(s)/Supplementary Readings
Lectures will be **self-contained,** and the students are expected to **make detailed course notes**. At various junctures, the instructor will point to selected sections in the references and upload relevant notes/handouts/slides. References (listed chronologically in time): • A. Beck, **First-Order Methods in Optimization**, SIAM, (2017) • D. P. Bertsekas, **Convex Optimization Algorithms**, Athena Scientific, Belmont, MA, (2015) • A. Beck, **Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB**, SIAM, (2014) • D. P. Palomar & Y. C. Eldar, (eds), **Convex optimization in signal processing and communications**, Cambridge University Press, (2010) • D. P. Bertsekas, **Convex Optimization Theory**, Athena Scientific, Belmont, MA, (2009) • J. Nocedal & S. J. Wright, **Numerical optimization**, second edition, (2006) [E-book available with Springer Access] • A. Ruszczynski, **Nonlinear Optimization**, Princeton University press, (2006) • S. Boyd & L. Vandenberghe, **Convex Optimization**, Cambridge University Press, (2004) [E-book available online]

Academic Honesty
The principles of truth and honesty are recognized as fundamental to a community of teachers and students. This means that all academic work will be done by the student to whom it is assigned without unauthorized aid of any kind. Plagiarism, cheating and other forms of academic dishonesty are prohibited. Any instances of academic dishonesty in this course (intentional or unintentional) will be dealt with swiftly and severely. Potential penalties include receiving a failing grade on the assignment in question or in the course overall. For further information, students should make themselves familiar with the relevant section of the LUMS student handbook.

Harassment Policy
SSE, LUMS and particularly this class, is a harassment free zone. There is absolutely zero tolerance for any behavior that is intended or has the expected result of making anyone uncomfortable and negatively impacts the class environment, or any individual's ability to work to the best of their potential. In case a differently abled student requires accommodations for fully participating in the course, students are advised to contact the instructor so that they can be facilitated accordingly. If you think that you may be a victim of harassment, or if you have observed any harassment occurring in the purview of this class, please reach out and speak to me. If you are a victim, I strongly encourage you to reach out to the Office of Accessibility and Inclusion at oai@lums.edu.pk or the sexual harassment inquiry committee at harassment@lums.edu.pk for any queries, clarifications, or advice. You may choose to file an informal or a formal complaint to put an end to offending behavior. You can find more details regarding the LUMS sexual harassment policy here. To file a complaint, please write to harassment@lums.edu.pk.

SSE Council on Equity and Belonging
In addition to LUMS resources, SSE's **Council on Belonging and Equity** is committed to devising ways to provide a safe, inclusive and respectful learning environment for students, faculty and staff. To seek counsel related to any issues, please feel free to approach either a member of the council or email at cbe.sse@lums.edu.pk.

Rights and Code of Conduct for Online Teaching
A misuse of online modes of communication is unacceptable. TAs and Faculty will seek consent before the recording of live online lectures or tutorials. Please ensure if you do not wish to be recorded during a session to inform the faculty member. Please also ensure that you prioritize formal means of communication (email, LMS) over informal means to communicate with course staff.

Prepared and Revised by:	**Prof. Hassan Mohy-ud-Din**
Revision Date:	**January 16, 2024**