EE 563: Convex Optimization

Spring 2023

Lecture 1 – Optimization As We Know It

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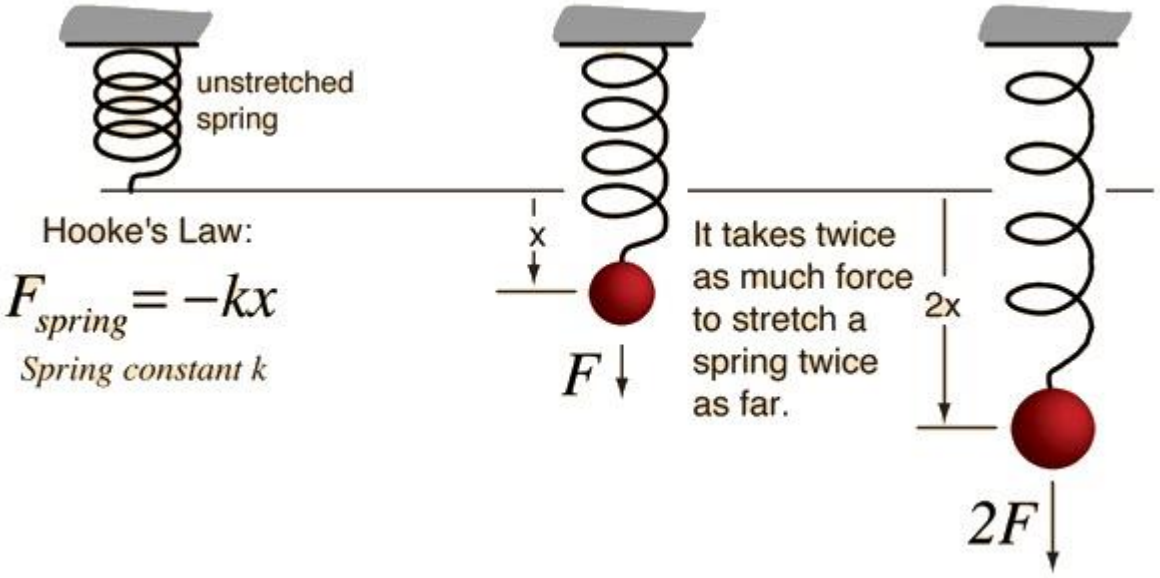
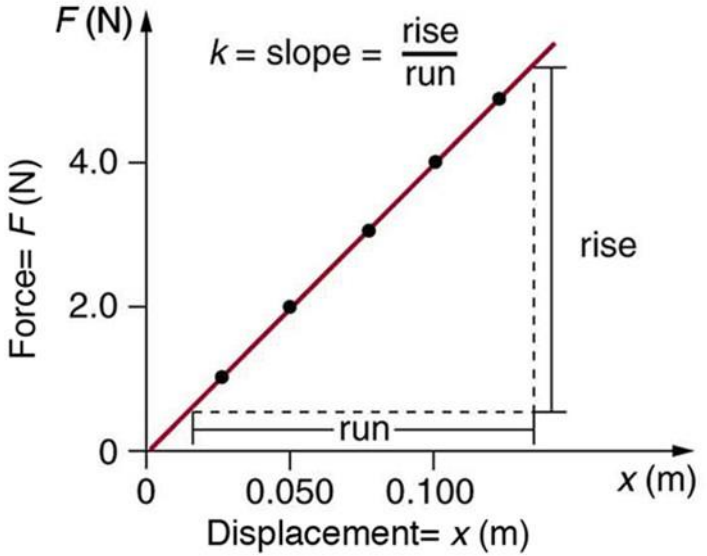
Story 1: Hooke's Law – Experiment

$$F = -kx$$

where k is the spring constant and x is the extension





m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

https://phys.org/news/2015-02-law.html
https://opentextbc.ca/physicstestbook2/chapter/hookes-law-stress-and-strain-revisited/

Hooke's Law – Curve Fitting

Line of best fit: How to define it?

Error criteria?

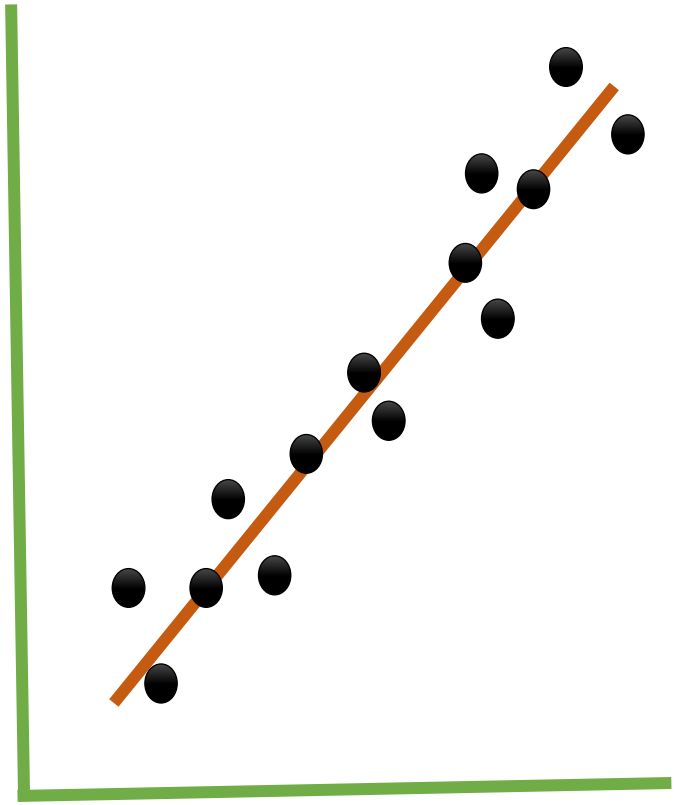
How to draw it?

Fitting function: Linear function: $y = mx + c$

Polynomial: $y = a_n x^n + a_{n-1} x^{n-1} + \cdots$

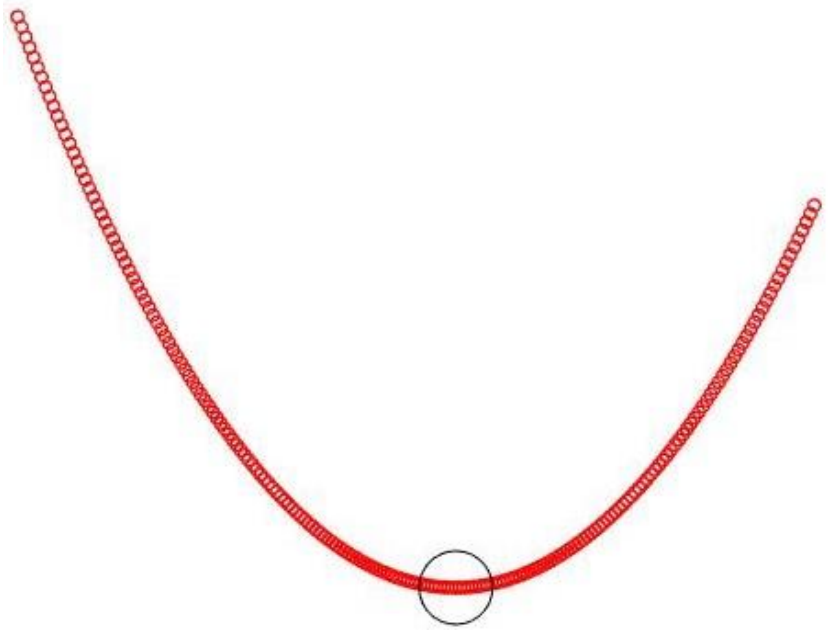
Error criteria: Least squares?

Constrained optimization: $c = 0$ – pass through the origin



Story 2: Extrema of Functions

Given: $f(x) = 3x^2 - 6x + \frac{9}{4}$



Stationary point: $f'(x) = 6x - 6 = 0 \rightarrow x = 1$

Minimum or Maximum: $f''(x) = 6 > 0 \implies \text{minimum}$

Story 3: System of Equations

Linear system of equations: $Ax = b$ e.g. convolution

Solution: $x = A^{-1}b$

Assumption: A is invertible (non-singular) $\Longrightarrow$ A is a square matrix

A is a well-conditioned matrix i.e. $\kappa = 1$ (ideally)

A can be stored on a computing machine

Goal: Find x that minimizes (squared) error aka discrepancy $e = \|Ax - b\|^2$

Need: An iterative approach

Components of an Iterative Algorithm

Physical Modeling: – Data
– Model
– Constraints
– Loss function

Algorithm: – Initialization
– Direction
– Step size
– Stopping criteria