Interval Forecasting of Hourly Electricity Spot Prices using Rolling Window Based Gaussian Process Regression

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***Abstract*—Electricity price forecasting is important to the energy companies in planning and decision making. Gaussian process (GP) regression is a powerful tool for probabilistic forecasts of time series data. In this paper, we employ GP regression for prediction interval (PI) based forecasting of electricity spot prices. At each hour of the day, a new parameter set is computed incorporating most recent available electricity price data. We compare performance of several kernels. Likelihood ratio (LR) test statistics are used to measure goodness of the out-of-sample forecasts. Results show that our scheme outperforms other schemes in literature. In one case, LR statistics are slightly better for an existing quantile regression averaging (QRA) based scheme .But QRA scheme employs 12 other forecasting schemes followed by performing regression on the forecasts by those 12 schemes. However, our results significantly better than other individual forecasting schemes such as ARX/SNARX and averaging schemes such as SIMPLE/LAD.**

***Keywords*—Prediction interval, electricity price, Gaussian process regression**

I. Introduction

Liberalization and deregulation of electricity markets has resulted in competitive and highly volatile electricity prices. Electricity prices have significant impact on all market participants including end consumers as well as electricity retailers. Due to deregulation of electricity markets, electricity price forecasting is critically important for optimized operation of power systems. Forecasting is also important for electricity retailers, particularly, due to the high interest in energy arbitrage with recent advanced and improved energy storage technologies. Arbitrage profit can be greatly increased by electricity price forecasting as knowledge of future electricity prices enables optimized scheduling of energy storage technologies. A precise forecasting scheme is helpful in developing a useful demand response program for efficient operation of power systems. Forecasting can also help retailers to change their bidding strategies for maximizing the profit.

Mismatch between supply and demand, intermittent and variable generation from renewable energy sources such as wind and solar, and power plants breakdown result in highly volatile electricity prices having sudden spikes and jumps. Therefore, modeling and forecasting of electricity prices are challenging.

In recent years, research in electricity price forecasting has gotten a tremendous attention and several forecasting schemes have been proposed [1]–[3]. Time series modeling is most widely used approach for forecasting of electricity prices. Time series based schemes use historic prices and exogenous variables such as load, time of the day and temperature [3], [4]. Two basic time series models are auto-regressive (AR) models and auto-regressive-moving-average (ARMA) [5]. Apart from these two basic models, a number of extension of these models have also been proposed for forecasting time series data that include auto-regressive-integrated-moving-average models (ARIMA) [6], [7] , AR models with exogenous variables models (ARX) and ARMA model with exogenous ARMAX [8]. Kristiansen [1] uses AR model for forecasting of electricity prices in Nord pool power market. The model captures and tracks seasonal variations from historical price data. In [9], clustering based methodology has been used to forecast day-ahead electricity load at household level. Nogales *et al.* [10] develop a regression model based on historical price data and electricity demand data. Their model demonstrates a reasonable accuracy for forecasting of electricity prices in California and Spanish power markets. In [11], a weighted nearest neighbor model is used for forecasting of electricity prices. Performance of the mentioned price forecasting models may deteriorate due to unexpected spikes in the prices.

Hybrid models are used to capture different patterns in the price data and may improve results [12]. Shafie-Khah *et al.* [13] propose a hybrid model based on ARIMA, wavelet transform and neural networks, for forecasting of day-ahead electricity prices. In [14], the proposed hybrid model is based on ARIMA, wavelet transform and support vector machine (SVM). The model is used for forecasting of electricity prices in the Australian national electricity market and New South Wales power markets. The hybrid model in [15] is based on particle swarm optimization and a fuzzy inference system and

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is used for forecasting electricity prices in the Spanish power market. A comprehensive review of probabilistic electricity price forecasting can be found in [16].

The objective of this paper is to explore usefulness of using GP regression for short term interval forecasting of electricity prices. We have compared performance of different covariance functions excluding ARD (*Automatic Relevance Determination*) based covariance functions due to a large number of hyperparameters and high computation time. We have chosen predictor variables same as chosen in [3], [4]. We have also used same dataset of electricity prices for accurately comparing results with these schemes. This paper makes two main contributions. First, the paper proposes GP regression based forecasting model for electricity prices which outperforms other individual and averaging based forecasting schemes. The second contribution is to compare performance of non-ARD based covariance functions.

The paper is organized as follows. In section II, we describe proposed GP regression based model, predictor variables and calibration details. Section III provides details about the dataset, experimental results and comparison with existing schemes in literature. Finally, section IV concludes this paper.

II. Proposed Method

A. Gaussian Process Regression

In this section, we give a brief overview of the fundamentals of GP regression. An interested reader is referred to the text [17] for detailed description. Intuitively, one can think of a GP as a distribution over functions. A GP is a collection of random variables, any finite number of which have a joint Gaussian distribution [17]. A GP $f(\mathbf{x})$ can be specified completely by a mean function $m(\mathbf{x})$ and a covariance function (or kernel) $k(\mathbf{x_1}, \mathbf{x_2})$ and is written as in eq. 1.

$$f(\mathbf{x}) \sim \mathrm{GP}\left(m(\mathbf{x}), k(\mathbf{x_1}, \mathbf{x_2})\right), \tag{1}$$

where dimension of $\mathbf{x}$,$\mathbf{x_1}$ and $\mathbf{x_2}$ is equal to the number of predictors d.

Suppose we have n_r training examples $(X, \mathbf{y})$ and n_t testing examples $(X', \mathbf{y}')$, where $X(X')$ is the matrix of dimension $n_r(n_t) \times d$ and $y(y')$ is the column vector of length $n_r(n_t)$. In GP regression, dependent variable y is assumed to be the value of the GP at $\mathbf{x}$ and is denoted by $f(\mathbf{x})$. By denoting y and y' by $f(X)$ and $f(X')$ respectively, joint distribution of $f(X)$ and $f(X')$ is jointly Gaussian as shown in eq. 2.

$$\begin{bmatrix} f(X) \\ f(X') \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} m(X) \\ m(X') \end{bmatrix}, \begin{bmatrix} k(X,X) & k(X,X') \\ k(X',X) & k(X',X') \end{bmatrix}\right). \tag{2}$$

Given X, X' and $f(X)$ and assuming Gaussian prior on f, conditional distribution of $f(X')$ is jointly Gaussian with mean

$$\mathbf{m} = m(X') + k(X', X)\, k(X, X)^{-1} \left(f(X) - m(X)\right) \tag{3}$$

and covariance matrix

$$C = k(X', X') - k(X', X)\, k(X, X)^{-1} k(X, X'). \tag{4}$$

TABLE I

KERNEL FUNCTIONS STUDIED IN THE PROPOSED SCHEME

Kernel Name	Kernel Function
Exponential	$k^{EXP}(r) = \sigma^2 \exp\left(-\frac{r}{l}\right)$
Squared exponential	$k^{SQEXP}(r) = \sigma^2 \exp\left(-\frac{r^2}{2l^2}\right)$
Matern32	$k^{M32}(r) = \sigma^2 \left(1 + \frac{\sqrt{3}r}{l}\right) \exp\left(-\frac{\sqrt{3}r}{l}\right)$
Matern52	$k^{M52}(r) = \sigma^2 \left(1 + \frac{\sqrt{5}r}{l} + \frac{5r^2}{3l^2}\right) \exp\left(-\frac{\sqrt{5}r}{l}\right)$
Rational quadtric	$k^{RQ}(r) = \sigma^2 \left(1 + \frac{r^2}{2\alpha l^2}\right)^{-\alpha}$

In our case, dimension of X' is one $(n_t = 1)$, therefore $\mathbf{m}$ and C reduce to scalars. Prediction about y' is made using the conditional distribution of $f(X')$ and is described in the subsection II-C.

B. Covariance Functions

Covariance function $k(X, X')$ specifies similarity between it's inputs and is used as a measure of covariance between $f(X)$ and $f(X')$. A covariance function is usually a function of the distance $r = (\mathbf{x_1} - \mathbf{x_2})^T (\mathbf{x_1} - \mathbf{x_2})$ between its inputs. Covariance functions have a set of parameters θ. For example, covariance functions studied in the present paper have two parameters, character length scale l and the signal standard deviation σ. Character length scale controls sensitivity of the covariance function with respect to the distance between the inputs. Smaller value of l means covariance function is sensitive to small distance between the inputs. Whereas larger value of l makes covariance function insensitive to small distance. The signal standard deviation is the overall scaling of the covariance value. Covariance functions which use separate characteristic length scale for each of the predictors are called ARD (*Automatic Relevance Determination*) kernels. In this study, we did not use ARD type kernels due to large number of parameters. Covariance functions used in this study are shown in table I.

C. Rolling Window based Calibration and PI Forecasting

Suppose, historical hourly price data $\{\, p_h^k : k = 1, 2, ..., N;\ h = 1, 2, ..., 24 \,\}$ for past N days are used to forecast the price at hour h of $(N+1)_{th}$ day, where p_h^k represents hourly spot price at hour h of k_{th} day. For forecasting of price at hour h $(h = 1, 2, 3, ..., 24)$ of the $(N+1)_{th}$ day, $\{\, p_h^k : k = 1, 2, 3, ..., N \,\}$ are used to calibrate GP regression model. The trained model is used to compute forecast price $\hat{p}_h^{N+1}$. Similarly, for each hour h $(h = 1, 2, 3, ..., 24)$ of the $(N+2)_{th}$ day, $\{\, p_h^k : k = 2, 3, 4, ..., N+1 \,\}$ are used to calibrate GP model which is then used to compute forecast price $\hat{p}_h^{N+2}$, and so on. The idea of using separate parameter set for each of the forecast is inspired from the work of Weron and Misiorek [4] who compared performance of 12 time series based models for price forecasting. We have selected the same predictor variables which are selected in [4] due to their good correlation with the output variable. Predictor variables are describe below.

- p_{t-24}: Price at 24 hour lag; the price at same hour yesterday
- p_{t-168}: Price at 168 hour lag; the price at same hour a week ago
- mp_t: Minimum of yesterday 24 hour prices
- z_t: Temperature
- D: Categorical variable representing day of the week

Each of the predictor variables except categorical variables are normalized in the range 0.01 to 1.0, log transformed and standardized to have mean zero and variance one before GP regression calibration. Forecasting is made in the transformed domain and forecast values are inverse transformed. Suppose μ_{ltn} σ_{ltn} are mean and variance of the conditional normal distribution obtained using GP regression from eqs. 3 and 4 in the log domain and suppose m and s are respectively the mean and the standard deviation used to standardize the data. First, we apply inverse operation of standardization on μ_{ltn} and σ_{ltn} using the mean m and the standard deviation s. We denote resulting mean and standard deviation by μ_{ln} and σ_{ln} respectively. Next, we obtain mean μ_n and variance σ_n of the normal distribution using eqs. 5 and 6 respectively which transform mean and variance of a normal distribution to mean and variance of a lognormal distribution [18].

$$\mu_n = e^{\mu_{ln} + \frac{\sigma_{ln}^2}{2}}, \tag{5}$$

$$\sigma_n^2 = e^{2\mu_{ln} + 2\sigma_{ln}^2} - e^{2\mu_{ln} + \sigma_{ln}^2}. \tag{6}$$

We obtain final μ and standard deviation σ by re-scaling μ_n and σ_n in the original range. Lower bound (L) and upper bound U of the PI with confidence level $1-\alpha$ can be computed using eq. 7.

$$[L, U] = \mu \pm z_{\alpha/2}\sigma, \tag{7}$$

where area under the standard normal curve to the left of $z_{\alpha/2}$ is $\frac{\alpha}{2}$.

D. Likelihood Ratio Test for Coverage of Interval Forecast

We use likelihood ratio (LR) tests [19] for evaluating goodness of the interval forecast. In this subsection we give a brief overview of the LR framework for testing the coverage of interval forecasts. The reader may find complete detail of LR framework in [19]. Suppose $\{(L_t,\ U_t):\ t = 1, 2, 3, ..., T\}$ is an out-of-sample interval forecast of the sequence $\{x_t:\ t = 1, 2, 3, ..., T\}$ with coverage probability θ, where L_t and U_t are the lower and upper limits of the interval forecast, respectively. The indicator variable I_t for the interval forecast $\{(L_t,\ U_t):\ t = 1, 2, 3, ..., T\}$ is defined in eq. 8 as [19]

$$I_t = \begin{cases} 1, & \text{if } x_t \in [L_t,\ U_t] \\ 0, & \text{if } x_t \notin [L_t,\ U_t] \end{cases} \tag{8}$$

A general criteria for testing the efficiency of an interval forecast is that the expected value of the indicator variable is equal to θ [19], i.e. $E(I_t) = \theta$. Furthermore, Christoffersen [19] proves that testing of the hypothesis that $E(I_t) = \theta$ is equivalent to testing of the hypothesis that the indicator variables I_t are independent Bernoulli random variables with parameter θ. Therefore, the testing correctness of the interval forecast reduces to testing that I_t have Bernoulli distributions.

Unconditional LR test is to test the hypothesis $E(I_t) = \theta$ against the alternative $E(I_t) \neq \theta$. The likelihood under the null and alternative hypotheses are

$$L(\theta) = \theta^{n_1}(1-\theta)^{n_0} \tag{9}$$

and

$$L(\alpha) = \alpha^{n_1}(1-\alpha)^{n_0} \tag{10}$$

respectively, where n_0 and n_1 are the number of zeros and number of ones in the indicator variable sequence and $\alpha = \frac{n_1}{n_0+n_1}$. Finally, the hypothesis can be tested using the standard likelihood ratio test using the test statistic

$$\text{LR}_{\text{unconditional}} = -2\log\left(\frac{L(\theta)}{L(\alpha)}\right) \tag{11}$$

which has a $\chi^2(1)$ distribution.

Although, unconditional coverage test tests the coverage of the interval forecast but it does not test whether the zeros and ones come together in a dependent manner. Conditional coverage test collectively tests the dependency of zeros and ones as well as the coverage of the interval forecast. In this case the null hypothesis of the unconditional coverage test is tested against the alternative that the sequence is dependent. Conditional coverage test can also be tested using the likelihood ratio test using the test statistic

$$\text{LR}_{\text{conditional}} = -2\log\left(\frac{L(\theta)}{L(\Pi)}\right) \tag{12}$$

which has a χ^2 distribution with 2 degrees of freedom and $L(\Pi)$ is defined as

$$L(\Pi) = \pi_{01}^{n_{01}}\pi_{11}^{n_{11}}(1-\pi_{01})^{n_{00}}(1-\pi_{11})^{n_{10}} \tag{13}$$

where $\pi_{ij} = \text{Prob}\left(I_t = j | I_{t-1} = i\right)$ and n_{ij} is the number of consecutive pairs ij in the indicator variable sequence.

We compute conditional and unconditional LR statistics for each of the hour h $(= 1, 2, 3, ..., 24)$ separately and calculate the number of hours for which the null hypotheses are rejected for unconditional and conditional coverage tests.

III. Results

Proposed GP process regression is implemented on Windows 10 using MATLAB R2017a. We use day-ahead hourly locational marginal prices (LMP) of electricity for JCPL zone of the PJM interconnection for the period Dec 24, 2010 to Jan 14, 2012. Electricity prices and temperature data are shown in the figure 1. As shown in the figure, Dec 24, 2010 to Sep 22, 2011 ($N = 273$ days) is the calibration period for forecasting prices on Sep 23, 2011. Similarly, for forecasting of prices on Sep 24, 2011, the calibration window is shifted to Dec 25, 2010 to Sep 23, 2011 and so on. We calculated predication intervals (PI) and compared nominal coverage to the true coverage. Coverage and other statistical properties of the PIs width for the forecast period by our scheme and the

TABLE II
UNCONDITIONAL COVERAGE OF 50% AND 90% PIS OF THE EXISTING SCHEMES (TOP FIVE ROWS) AND THE PROPOSED SCHEME (BOTTOM FIVE ROWS)

	Nominal Coverage: 50%					Nominal Coverage: 90%				
	Coverage	Mean	Median	Std Dev	IQR	Coverage	Mean	Median	Std Dev	IQR
ARX	69.74	8.63	8.66	3.33	5.25	96.13	21.28	21.34	8.29	13.02
SNARX	56.51	6.09	5.94	2.64	4.21	94.23	20.73	20.64	8.78	15.28
SIMPLE	58.63	6.32	5.89	2.89	5.77	94.44	25.73	23.22	15.74	25.86
LAD	56.36	6.73	5.79	3.66	6.93	93.64	26.20	21.87	17.21	26.33
QRA	53.55	6.4	5.62	3.78	5.19	92.07	21.10	19.51	12.09	18.51
EXP	48.21	7.07	5.82	4.66	4.76	88.12	17.25	14.2	11.36	11.62
SQEXP	48.79	6.77	5.53	4.44	4.40	87.83	16.52	13.49	10.82	10.73
M32	48.83	6.81	5.54	4.50	4.49	88.08	16.60	13.51	10.97	10.96
M52	48.94	6.78	5.53	4.47	4.45	87.97	16.54	13.48	10.89	10.86
RQ	48.79	6.79	5.60	4.47	4.52	87.72	16.55	13.66	10.89	11.01

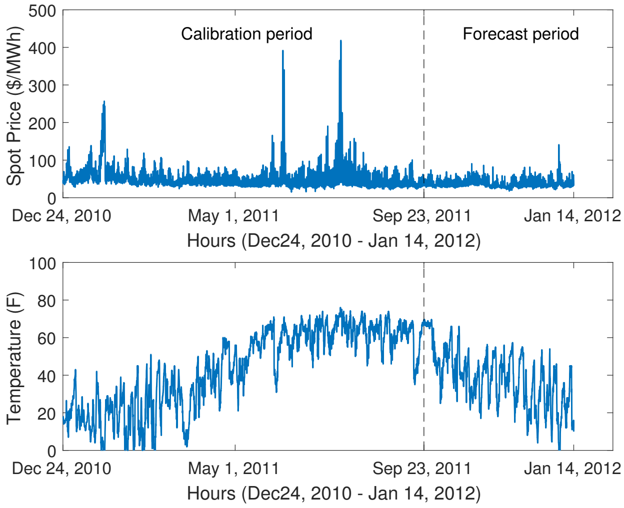

Fig. 1. Hourly electricity prices and temperature

TABLE III
NUMBER OF HOURS FOR WHICH NULL HYPOTHESIS IS REJECTED IN THE EXISTING SCHEMES (TOP FIVE ROWS) AND THE PROPOSED SCHEME (BOTTOM FIVE ROWS)

	Unconditional Coverage				Conditional Coverage			
	50% PI		90% PI		50% PI		90% PI	
α	1%	5%	1%	5%	1%	5%	1%	5%
ARX	20	21	14	17	20	20	24	16
SNARX	5	10	8	13	4	9	8	12
SIMPLE	8	11	13	13	7	10	14	16
LAD	9	11	11	12	9	13	11	14
QRA	2	6	2	3	2	3	3	5
EXP	6	8	4	5	6	10	3	5
SQEXP	6	10	3	6	8	12	6	7
M32	5	10	2	7	5	11	5	6
M52	6	10	2	6	5	11	5	7
RQ	3	8	2	7	4	9	5	8

schemes in [3], [4] are shown in the table II. PI coverage by the proposed scheme is closer to the true coverage for all the five kernels in case of 50% nominal coverage. Out of these five kernels, Matern52 (M52) has a coverage closest to the true coverage with Matern32 (M32) and squared exponential (SQEXP) respectively being the second and third kernels. Mean and Median of the PI widths are approximately the same with ARX having maximum mean/median PI width. Standard deviation of the PI width is minimum for SNARX scheme with proposed scheme having smaller standard deviation from ARX, SIMPLE, LAD and QRA schemes. In case of 90%, exponential, Matern32 and Matern 52 kernels have closest PI coverage with the nominal coverage. The mean/median of the PI widths for all the kernels used in the proposed scheme are smallest than mean/median of all other five schemes in literature.

Next, we apply LR test [19] to test the goodness of the PIs. These tests have also been applied for the schemes in [4] and for comparison we have adopted the same test. We compute two LR statistics for each of the unconditional and conditional coverage. LR for unconditional and conditional coverage are distributed as $\chi^2(1)$ and $\chi^2(2)$, respectively. We apply LR test for each of the 24 hours separately. Table III shows the number of hours for which the PI is rejected by the LR test at $\alpha = 1\%$ and $\alpha = 5\%$ level of significance. In all cases, ARX and QRA schemes have maximum and minimum number of hours for which PI is rejected, respectively. For all other schemes, our proposed scheme have minimum number of rejected PIs roughly having equal performances.

IV. CONCLUSION

In this paper, we have investigated GP regression for short term interval based forecasting of spot electricity prices and compared performance of five kernels. Results indicate EXP and RQ kernels outperform the other three kernels and have roughly comparable performance. Comparing with existing schemes in literature, proposed scheme outperforms all other schemes except QRA whose results are slightly better than ours. However, the results of nominal coverage and true coverage suggest that proposed scheme outperforms all other schemes including QRA. Slight better performance of QRA scheme is owing to the fact that it is an averaging scheme and employs 12 other time series based forecasting schemes in making final forecast. Nonetheless, performance of proposed scheme is comparable to QRA scheme even if proposed scheme is an individual forecasting scheme. Proposed scheme highly outperforms the individual schemes ARX/SNARX, and the averaging schemes SIMPLE/LAD. Comparing higher number of parameters and calibration of 12 models for a single forecast in case of QRA and a forecast with a single model with a comparable performance also favors proposed scheme over QRA. In the future, we aim to employ proposed scheme in QRA and other averaging schemes to investigate the performance gain.

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